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MATDIP401

Fourth Semester B.E. Degree Examination, Jan./Feb.2021
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. If l, m, n are the direction cosines of a line, then show that $l^2 + m^2 + n^2 = 1$. **(07 Marks)**

b. A line makes an angle α, β, γ, δ with diagonals of a cube. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$. **(07 Marks)**

c. Find the angle between the lines AB and CD where $A = (3, 4, 5)$, $B = (4, 6, 2)$, $C = (-1, 2, 4)$ and $D = (1, 0, 5)$. **(06 Marks)**

2 a. Find the equation of a plane posses through the points $(3, -3, 1)$ and perpendicular to the planes $7x + y + 2z = 6$ and $3x + 5y - 6z = 8$. **(07 Marks)**

b. Find the angle between two planes, $x - y + 2z - 9 = 0$ and $2x + y + z = 7$. **(07 Marks)**

c. Find the shortest distance between the lines, $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ **(06 Marks)**

3 a. Prove that $\vec{a} \times \left(\vec{b} + \vec{c}\right) + \vec{b} \times \left(\vec{c} + \vec{a}\right) + \vec{c}\left(\vec{a} + \vec{b}\right) = 0$. **(07 Marks)**

b. Find the unit normal to both the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Also find sign of the angle between them. **(07 Marks)**

c. Show that the vectors $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{B} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{C} = 3\hat{i} + 4\hat{j} - \hat{k}$ are coplanar. **(06 Marks)**

4 a. Find the unit tangent vector of the space curve, $\vec{r} = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$. **(07 Marks)**

b. A particle moves along the curve, $x = (1 - t^3)$, $y = (1 + t^2)$ and $z = (2t - 5)$. Determine the velocity and acceleration. **(06 Marks)**

c. Find the angle between the surfaces, $x^2 + y + z^2 = 9$ and $x = z^2 + y^2 - 3$ at $(2, -1, 2)$. **(07 Marks)**

5 a. Find the directional derivative of x^2yz^3 at $(1, 1, 1)$ in the direction of $i + j + 2k$. **(07 Marks)**

b. Prove that $F.\text{curl}F = 0$ for $F = (x + y + 1)i + j - (x + y)k$. **(07 Marks)**

c. Show that the vectors, $\vec{F} = (2xy + z^2)\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 2zx)\hat{k}$ is irrotational. **(06 Marks)**

6 a. Find the Laplace transform of t^n, where 'n' is a positive integer. **(07 Marks)**

b. Find $L[\cos t \cos 2t \cos 3t]$. **(07 Marks)**

c. Find $L\left[\frac{\sin^2 t}{t}\right]$. **(06 Marks)**

7 a. Find $L^{-1}\left[\frac{s+5}{s^2-6s+13}\right]$. **(07 Marks)**

b. Find $L^{-1}\left[\frac{1}{(s+1)(s+2)(s+3)}\right]$. **(07 Marks)**

c. Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$. **(06 Marks)**

8 a. Using Laplace transform, solve
$y'' + 4y' + 4y = e^{-t}$; $y(0) = 0$, $y'(0) = 0$. **(10 Marks)**

b. By using Laplace transformations, solve the differential equation, $y'' + 4y' + 3y = 0$, subject to the condition $y(0) = 0$ and $y'(0) = 1$. **(10 Marks)**

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